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On P-wave meson decay constants in the heavy quark limit of QCD

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Abstract

In previous work it has been shown that, either from a sum rule for the subleading Isgur-Wise function $\xi_3(1)$ or from a combination of Uraltsev and Bjorken SR, one infers for P-wave states $|\tau_{1/2}(1)| \ll |\tau_{3/2}(1)|$. This implies, in the heavy quark limit of QCD, a hierarchy for the *production* rates of P-states $\Gamma(\bar{B}_d \to D\left(\frac{1}{2}\right)\ell\nu) \ll \Gamma(\bar{B}_d \to D\left(\frac{3}{2}\right)\ell\nu)$ that seems at present to be contradicted by experiment. It was also shown that the decay constants of $j = \frac{3}{2}$ P-states vanish in the heavy quark limit of QCD, $f_{3/2}^{(n)} = 0$. Assuming the *model* of factorization in the decays $\bar{B}_d \to \bar{D}_s^{**} D$, one expects the opposite hierarchy for the *emission* rates $\Gamma(\bar{B}_d \to \bar{D}_s\left(\frac{3}{2}\right)D) \ll \Gamma(\bar{B}_d \to \bar{D}_s\left(\frac{1}{2}\right)D)$, since $j = \frac{1}{2}$ P-states are coupled to vacuum. Moreover, using Bjorken SR and previously discovered SR involving heavy-light meson decay constants and IW functions, one can prove that the sums $\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2$, $\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2$ (where $f^{(n)}$ and $f_{1/2}^{(n)}$ are the decay constants of S-states and $j = \frac{1}{2}$ P-states) are divergent. This situation seems to be realized in the relativistic quark models à la Bakamjian and Thomas, that satisfy HQET and predict decays constants $f^{(n)}$ and $f_{1/2}^{(n)}$ that do not decrease with the radial quantum number n.

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In previous work [1], [2] we have pointed out a problem between experiment and the heavy quark limit of QCD for the semi-leptonic decays of B mesons to $L = 1$ excited states $D_{0,1}\left(\frac{1}{2}\right)$, $D_{1,2}\left(\frac{3}{2}\right)$. In a few words, the argument is as follows.

From Bjorken SR [3] [4]

$$\rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}(1)|^2 \qquad (1)$$

and the recently demonstrated Uraltsev SR [5]

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4} \quad . \qquad (2)$$

one obtains,

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 = \frac{\rho^2}{3} \qquad (3)$$

$$\sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{3}\left(\rho^2 - \frac{3}{4}\right) \quad . \qquad (4)$$

One can see that $\sum\limits_n |\tau_{3/2}^{(n)}(1)|^2$ is proportional to ρ^2 and that $\sum\limits_n |\tau_{1/2}^{(n)}(1)|^2$ is proportional to the *deviation* of ρ^2 from the lower bound $\frac{3}{4}$. Then, there is little room left for $\sum\limits_n |\tau_{1/2}^{(n)}(1)|^2$, as it has been pointed out recently from a SR obtained for the subleading function $\xi_3(1)$ [3]. The expected hierarchy for the form factors $|\tau_{3/2}(1)| \gg \tau_{1/2}(1)$, that can be inferred from the precedent equations, implies that $\bar{B}_d \to D^{**}\ell\nu$ and $\bar{B}_d \to D^{**}\pi$ (assuming factorization, a reasonable assumption in view of the recent papers on non-leptonic decays with emission of a light meson) are dominated by the narrow resonances

$$\Gamma\left(\bar{B}_d \to D_{1,2}\left(\tfrac{3}{2}\right)\ell\nu\right) \gg \Gamma\left(\bar{B}_d \to D_{0,1}\left(\tfrac{1}{2}\right)\ell\nu\right) \qquad (5)$$

$$\Gamma\left(\bar{B}_d \to D_{1,2}\left(\tfrac{3}{2}\right)\pi\right) \gg \Gamma\left(\bar{B}_d \to D_{0,1}\left(\tfrac{1}{2}\right)\pi\right) \quad . \qquad (6)$$

On the other hand, it was demonstrated that the decay constants of $j = \frac{3}{2}$ P-wave mesons vanish, $f_{3/2}^{(n)} = 0$, in the heavy quark limit of QCD [6], [7]. This result can be summarized in the statement that we expect the "broad" D^{**} resonances $(j = \frac{1}{2})$ to have a much larger decay constant than the "narrow" ones $(j = \frac{3}{2})$. This to be contrasted to the opposite expected hierarchy for form factors, $|\tau_{3/2}| \gg |\tau_{1/2}|$

[1], that can be inferred from equations (3), (4). This hierarchy in the *production* is expected to be opposite to the one due to the selection rule $f^{(n)}_{3/2} = 0$,

$$\Gamma\left(\bar{B}_d \to \bar{D}_{s\ 1,2}\left(\tfrac{3}{2}\right) D\right) \ll \Gamma\left(\bar{B}_d \to \bar{D}_{s\ 0,1}\left(\tfrac{1}{2}\right) D\right) \tag{7}$$

where there is *emission* of $\bar{D}_{sJ}(j)$. Of course, this is only a qualitative statement, because factorization in the decays (7) is just a model and is not in a firm status as in the light meson emission case (6) [8]. To summarize, "broad" resonances ($j = \frac{1}{2}$) are expected to be suppressed in the production, while "narrow" resonances ($j = \frac{3}{2}$) are expected to be suppressed in the emission. The BaBar experiment has started looking at $B_d \to (\bar{D}K)D$ [9] where such decays with emitted excited states might be seen, but the statistics has to be improved.

Let us now make some further remarks on decay constants of excited states. Imposing duality to $\Delta\Gamma$ in the B^0_s-$\bar{B}^0_s$ system in the heavy b, c quark limit, the following sum rules have been demonstrated, in the heavy quark limit of QCD [6] :

$$\sum_n \frac{f^{(n)}}{f^{(0)}}\ \xi^{(n)}(w) = 1 \tag{8}$$

$$\sum_n \frac{f^{(n)}_{1/2}}{f^{(0)}}\ \tau^{(n)}_{1/2}(w) = \frac{1}{2} \tag{9}$$

The decay constants for S-states $f^{(n)}$ and for $j = \frac{1}{2}$ P-states $f^{(n)}_{1/2}$ are properly defined in ref. [6]. The sum rules (8)-(9) are strong constraints, as they hold for any value of w.

The w-dependence in eqns. (8), (9) was obtained by considering the two-body intermediate states of the type $D_s\bar{D}_s$ (ground state and excited states) for the diagrams of the spectator quark type. As explained in ref. [6], in this kind of diagrams the transition $\bar{B}_s \to D_s\bar{D}_s \to B_s$ occurs by $\bar{D}_s$ emission and D_s production by the $\bar{B}_s$ ($\bar{s}$ quark being spectator), followed by D_s emission and $\bar{D}_s$ production by the B_s. In the heavy quark limit, the expression for the contribution of one intermediate state is proportional to a quantity of the type $f^2_{D_s}[\xi(w_c)]^2$ where f_{D_s} is a generic ground state or excited D_s meson decay constant and $\xi(w_c)$ is a generic Isgur-Wise function $\xi^{(n)}(w)$, $\tau^{(n)}_{1/2}(w)$ or $\tau^{(n)}_{3/2}(w)$ [4], taken at the fixed value of $q^2 = m^2_c$:

$$w_c = \frac{m^2_b + m^2_c - q^2}{2m_b\ m_c} = \frac{m_b}{2m_c} \quad . \tag{10}$$

Varying the ratio m_b/m_c, the w-dependence in the IW functions appears. Identifying the sum over the exclusive modes with the contribution to the quark box diagram having the same topology [10], one obtains, considering the vector or the axial weak current, the sum rules (8) and (9).

In an earlier paper [11], we had demonstrated that indeed duality for $\Delta\Gamma_{B_s}$ occurs for $N_c = 3$ in the heavy quark limit in the particular Shifman-Voloshin conditions $\Lambda_{QCD} \ll m_b - m_c \ll m_b, m_c$ [12].

On the other hand, the sum rules (8), (9) were studied [13] within the relativistic quark models of the Bakamjian-Thomas (BT) type [14], that satisfy HQET relations for form factors and decay constants. The sum rules are satisfied for different values of w, although the numerical convergence becomes worst as w increases.

From (8), (9), using Schwartz inequality, one can obtain the lower bounds :

$$\left[\sum_n \left[\xi^{(n)}(w)\right]^2\right]\left[\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2\right] \geq \left(\sum_n \frac{f^{(n)}}{f^{(0)}}\xi^{(n)}(w)\right)^2 = 1 \tag{11}$$

$$\left[\sum_n \left[\tau_{1/2}^{(n)}(w)\right]^2\right]\left[\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2\right] \geq \left(\sum_n \frac{f_{1/2}^{(n)}}{f^{(0)}}\tau_{1/2}^{(n)}(w)\right)^2 = \frac{1}{4} \tag{12}$$

Considering first these inequalities at $w = 1$, one can see, from $\xi^{(n)}(1) = 0$ for $n \neq 0$, that (11) does not provide any useful constraint. However, from (12) at $w = 1$ and (4) one obtains the bound for the $j = \frac{1}{2}$ P-wave decay constants

$$\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2 \geq \frac{3}{4\rho^2 - 3} \tag{13}$$

that contains the IW slope ρ^2 in the right-hand side. Although the bound (13) is very weak, it deserves a few comments. Its r.h.s., and also its l.h.s. diverge as $\rho^2 \to \frac{3}{4}$ and it reflects the fact that the excited P-wave mesons $D_{0,1}(j = \frac{1}{2})$ do indeed couple to vacuum, respectively through the vector and axial currents for $J = 0, 1$, as was already proved from the sum rule (9). This is to be contrasted with the selection rule $f_{3/2}^{(n)} = 0$ [6] [7] that applies to the excited mesons $D_{1,2}(j = \frac{3}{2})$. In the example of the non-relativistic quark model, also given as an illustration in ref. [6], both sides of the inequality (13) are of $O(v^2/c^2)$, since $f_{1/2}^{(n)}$ is of $O(v/c)$ (formula (17) of [6]) and ρ^2 is of $O(c^2/v^2)$ (formula (52) of [6]). In this case, as can be easily seen using completeness, the l.h.s. of (13) is infinite, proportional to the divergent

integral $\int_0^\infty p^4 dp$, two powers of p coming from the current and p^2 from the measure $\mathrm{d}\vec{p}$.

Actually, one can demonstrate this divergence also in field theory, in the heavy quark limit of QCD by considering arbitrary large w. From Bjorken SR for any value of w [4],

$$\frac{w+1}{2}\sum_n |\xi^{(n)}(w)|^2 + (w-1)\left[2\sum_n |\tau_{1/2}^{(n)}(w)|^2 + (w+1)^2\sum_n |\tau_{3/2}^{(n)}(w)|^2\right] + \cdots = 1 \tag{14}$$

one obtains, since all terms in this sum are definite positive

$$\sum_n |\xi^{(n)}(w)|^2 \leq \frac{2}{w+1} \qquad \sum_n |\tau_{1/2}^{(n)}(w)|^2 \leq \frac{1}{2(w-1)} \tag{15}$$

From (11) and (12) for any w one gets

$$\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2 \geq \frac{w+1}{2} \qquad \sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2 \geq \frac{w-1}{2} \tag{16}$$

Since the l.h.s. of these inequalities is independent of w, that can be made arbitrarily large, one concludes that the sums $\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2$, $\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2$ must diverge.

This situation seems to be realized in quark models à la Bakamjian and Thomas. The decay constants $f^{(n)}$ and $f_{1/2}^{(n)}$ were computed within the BT quark models for different quark-antiquark potentials up to $n = 10$, and the convergence of the SR (8) and (9) was studied for different values of w [13]. The error on the decay constants induced by a truncation procedure in the calculation increases strongly with n. The decay constants $f^{(n)}$ and $f_{1/2}^{(n)}$ do not decrease with increasing n. For the most sophisticated Godfrey-Isgur potential, that describes all $q\bar{q}$, $q\bar{Q}$ and $Q\bar{Q}$ quarkonia, including spin dependent effects [15], one obtains the decay constants of the Table for the S-states and the $j = \frac{1}{2}$ P-states [13].

Radial excitation	$\sqrt{M} f^{(n)}$ (GeV$^{3/2}$)	$\sqrt{M} f_{1/2}^{(n)}$ (GeV$^{3/2}$)
$n = 0$	0.67(2)	0.64(2)
$n = 1$	0.73(4)	0.66(4)
$n = 2$	0.76(5)	0.71(5)
$n = 3$	0.78(9)	0.73(8)
$n = 4$	0.80(10)	0.76(11)

Table : Decay constants $f^{(n)}$, $f_{1/2}^{(n)}$ for radial excitations S-states and $j = \frac{1}{2}$ P-states in the GI spectroscopic model. The estimated error is in parentheses [13], and M is the bound state mass.

In view of the values of the decay constants in the Table, besides the qualitative hierarchy (7), one expects for the $n = 0$ states, assuming factorization, neglecting Penguin diagrams and also the mass differences between the P states and the ground state :

$$\begin{aligned}
&\frac{\Gamma(\bar{B}_d \to D_{s0}^{-}\left(\frac{1}{2}\right) D^+)}{\Gamma(\bar{B}_d \to D_s^- D^+)} \cong \frac{\Gamma(\bar{B}_d \to D_{s0}^{-}\left(\frac{1}{2}\right) D^{*+})}{\Gamma(\bar{B}_d \to D_s^- D^{*+})} \cong 1 \\
&\frac{\Gamma(\bar{B}_d \to D_{s1}^{-}\left(\frac{1}{2}\right) D^+)}{\Gamma(\bar{B}_d \to D_s^{*-} D^+)} \cong \frac{\Gamma(\bar{B}_d \to D_{s1}^{-}\left(\frac{1}{2}\right) D^{*+})}{\Gamma(\bar{B}_d \to D_s^{*-} D^{*+})} \cong 1 \ .
\end{aligned} \tag{17}$$

These relations follow from the approximate numerical equality $f^{(0)} \cong f_{1/2}^{(0)}$ in the Table and the heavy quark relations among, respectively, the decay constants of D, D^* and $D_0\left(\frac{1}{2}\right)$, $D_1\left(\frac{1}{2}\right)$ mesons [6].

In conclusion, we have underlined a disymmetry in the prediction of the rates of production and emission of P-wave heavy-light mesons, and we have undertaken a theoretical discussion of decay constants of excited heavy mesons.

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